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LATTICE RESULTS
ON GLUON AND GHOST PROPAGATORS IN LANDAU GAUGE

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Abstract. We present clear evidence of strong effects of Gribov copies in Landau gauge gluon and ghost propagators computed on the lattice at small momenta by employing a new approach to Landau gauge fixing and a more effective numerical algorithm. It is further shown that the new approach substantially decreases notorious finite-volume effects.

1 Introduction

The gauge-variant Green functions, in particular for the covariant Landau gauge, are important for various reasons. Their infrared asymptotics is crucial for gluon and quark confinement according to scenarios invented by Gribov [1] and Zwanziger [2] and by Kugo and Ojima [3]. They have proposed that the Landau gauge ghost propagator is infrared diverging while the gluon propagator is infrared vanishing. The interest in these propagators was stimulated in part by the progress achieved in solving Dyson-Schwinger equations (DSE) for these propagators (for a recent review see [4]). Recently it has been argued that a unique and exact power-like infrared asymptotic behavior of all Green functions can be derived without truncating the hierarchy of DSE [5]. This solution agrees completely with the scenarios of confinement mentioned above. The lattice approach is another powerful tool to compute these propagators in an *ab initio* fashion but not free of lattice artefacts. So far, there is no consensus between DSE and lattice results. For the gluon propagator, the ultimate decrease towards vanishing momentum has not yet been established in lattice computations. Lattice results for the ghost propagator qualitatively agree with the predicted diverging behavior but show a substantially smaller infrared exponent [6].

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The lattice approach has its own limitations. The effects of the finite volume might be strong at the lowest lattice momenta. Moreover, gauge fixing is not unique resulting in the so-called *Gribov problem*. Previously it has been concluded that the gluon propagator does not show effects of Gribov copies beyond statistical noise, while the ghost propagator has been found to deviate by up to 10% depending on the quality of gauge fixing [7, 8].

Recently a new, extended approach to Landau gauge fixing has been proposed [9]. In this contribution we present results obtained within this new method and using a more effective numerical algorithm for lattice gauge fixing, the simulated annealing (SA) algorithm. Results for the gluon propagator have been already discussed in [10], while results for the ghost propagator are presented here for the first time.

2 Computational details

Our computations have been performed for one lattice spacing corresponding to rather strong bare coupling, at $\beta \equiv 4/g_0^2 = 2.20$, on lattices from 8^4 up to 32^4. The corresponding lattice scale a is fixed adopting $\sqrt{\sigma}a = 0.469$ [11] with the string tension put equal to $\sigma = (440\ \mathrm{MeV})^2$. Thus, our largest lattice size 32^4 corresponds to a volume $(6.7\ \mathrm{fm})^4$.

In order to fix the Landau gauge for each lattice gauge field $\{U\}$ generated by means of a MC procedure, the gauge functional

$$F[g] = \frac{1}{2N_{links}} \sum_{x,\mu} \mathrm{tr}\left(\mathrm{g(x)U_{x\mu}g^{\dagger}(x+\hat{\mu})}\right) \qquad (1)$$

is iteratively maximized with respect to a gauge transformation $g(x)$ which is usually taken as a periodic field. In $SU(N)$ gluodynamics the lattice action and the path integral measure are invariant under extended gauge transformations which are periodic modulo $Z(N)$,

$$g(x + L\hat{\nu}) = z_{\nu} g(x)\,, \qquad z_{\nu} \in Z(N) \qquad (2)$$

in all four directions. Any such gauge transformation is equivalent to a combination of a periodic gauge transformation and a flip $U_{x\nu} \rightarrow z_{\nu}\, U_{x\nu}$ for a 3D hyperplane with fixed x_{ν}. With respect to the flip transformation all gauge copies of one given field configuration can be split into N^4 flip sectors. The traditional gauge fixing procedure considers one flip sector as a separate gauge orbit. The new approach suggested in [9] combines all N^4 sectors into one gauge orbit. Note, that this approach is not applicable in a gauge theory with fundamental matter fields because the action is not invariant under transformation (2), while in the deconfinement phase of $SU(N)$ pure gluodynamics it should be modified: only flips in space directions are left in the gauge orbit. In

practice, few Gribov copies are generated for each sector and the best one over all sectors is chosen by employing an optimized simulating annealing algorithm in combination with finalizing overrelaxation.

3 Results

Thus, we are looking for the gauge copy with the highest value of the gauge functional among gauge copies belonging to the enlarged gauge orbit as defined above. It is immediately clear that this procedure allows to find higher local maxima of the gauge functional (1) than the traditional ('old') gauge fixing procedures employing purely periodic gauge transformations and the standard overrelaxation algorithm. Obviously the two prescriptions to fix the Landau gauge, the traditional one and the extended one, are not equivalent. Indeed, for some modest lattice volumes and for the lowest momenta it has been shown in Ref. [9] that they give rise to different results for the gluon as well as the ghost propagators. Comparing results for different lattice sizes we found that the results seem to converge to each other in the large volume limit. It is important that results obtained with the new prescription converge towards the infinite volume limit much faster. In Fig. 1 the gluon propagator $D(p^2)$ is

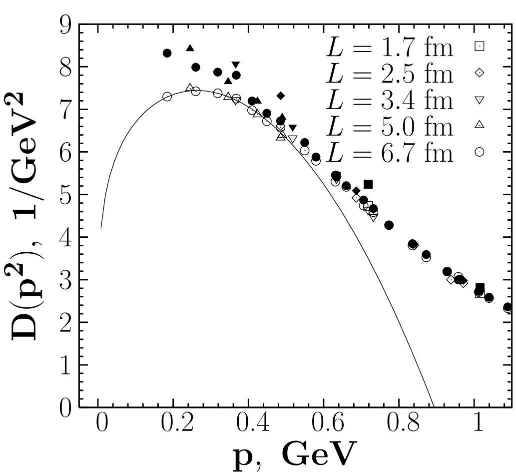

Figure 1: Comparison of the gluon propagator computed with new (empty symbols) and old (filled symbols) procedures on lattices of various sizes. Error bars which are in most cases smaller than symbol sizes are not shown for clarity. The curve corresponds to eq. (3).

shown.[b] One can see that the Gribov copy effects are strong up to $p \sim 0.6$ GeV. Furthermore, results obtained with the new procedure show no finite volume

[b]For definition of $D(p^2)$ as well as of the ghost dressing function $J(p^2)$ see Refs. [9, 10].

effects while these effects are clearly seen for results obtained with the old procedure. We have also checked, whether our result can be seen in agreement with the expectation $D(p \to 0) = 0$. We made a fit for $0 < p < 0.5$ GeV with the function

$$D(p^2) = (p^2)^{\alpha} \cdot (g_0 + g_1 \cdot p^2)\,. \tag{3}$$

and found $\alpha = 0.09(1)$ which is in qualitative agreement with the DSE result [4].

In Fig. 2 we show our results for the ghost dressing function $J(p^2)$. Again, strong Gribov copies effects are seen. On the other hand, the finite-volume effects are not strong for both procedures. We confirm earlier $SU(3)$ results [6] that the lattice ghost propagator is much less diverging in the infrared limit than it is predicted by DSE solutions [4]. Moreover, within the new procedure this disagreement becomes even stronger. This is one of the problems to be resolved in future studies.

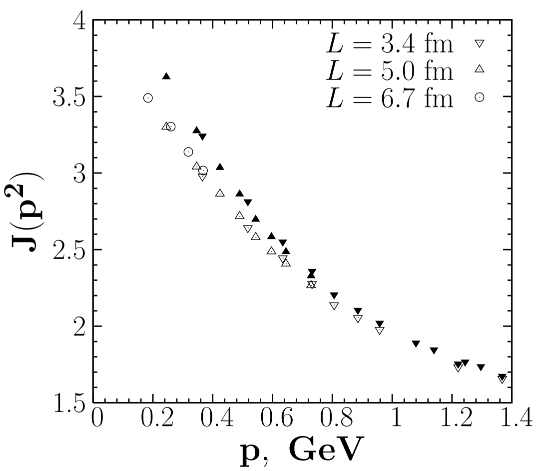

Figure 2: Ghost dressing function for two procedures. Symbols are as in Fig. 1.

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